Extrapolating Solid Earth Models to terrestrial exoplanets

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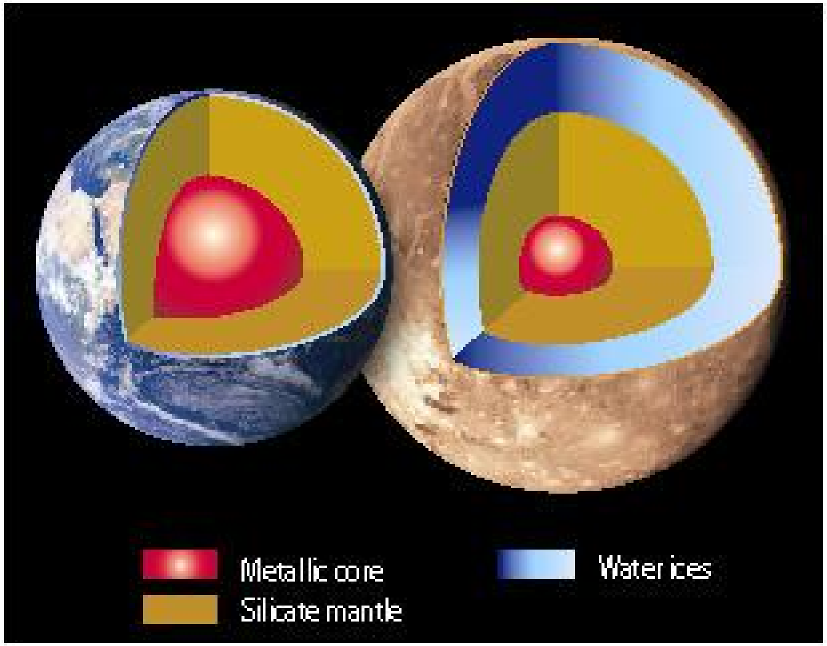
More and more planets are being discovered

Mass and radius are two parameters that can be measured

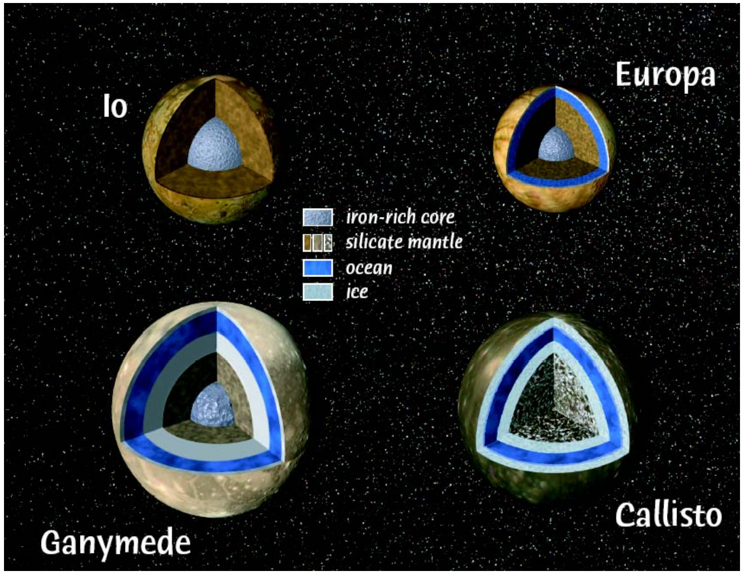
- Iron planets (Mercury)
- Terrestrial planets
- Ocean / Icy planets
    - Icy Moons
    - Uranus and Neptune without their atmosphere
- Giant planets



This talk will focus on terrestrial (Earth-like) planets and ocean (icy) planets

Extrapolating Solid Earth Models to terrestrial exoplanets

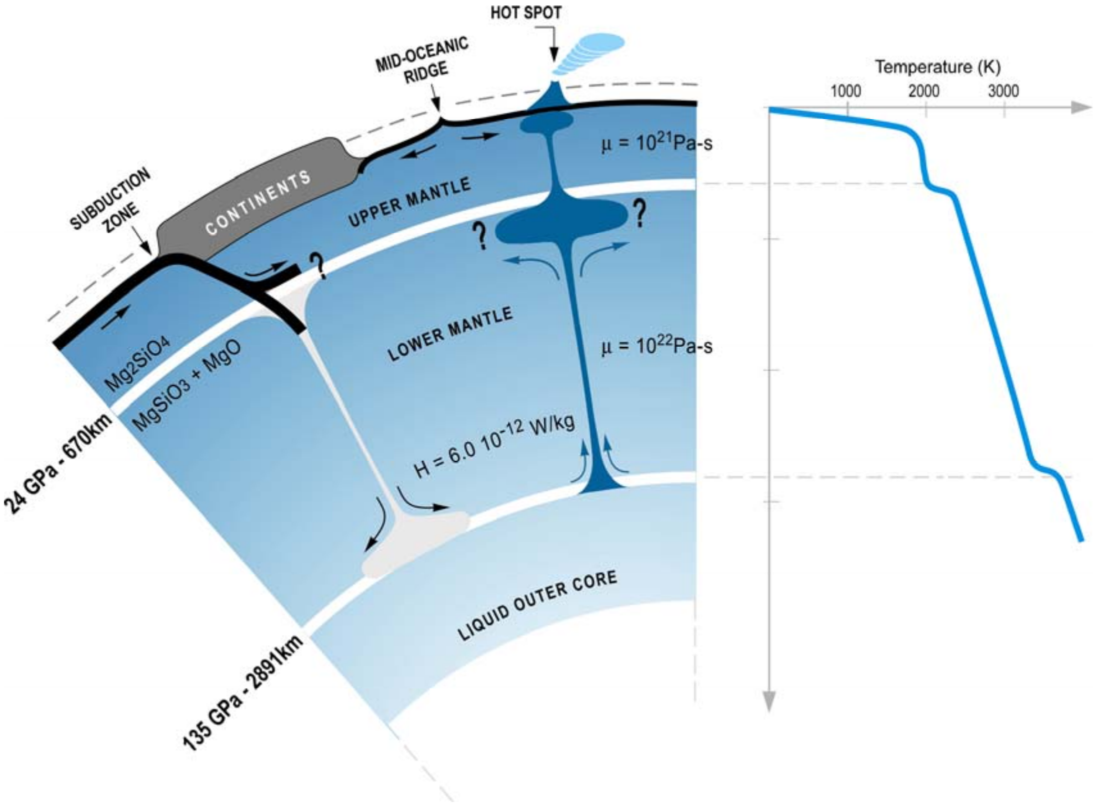
Information from Earth
Information from icy satellites
Phase diagrams
Heat production and heat transfer : temperature profile

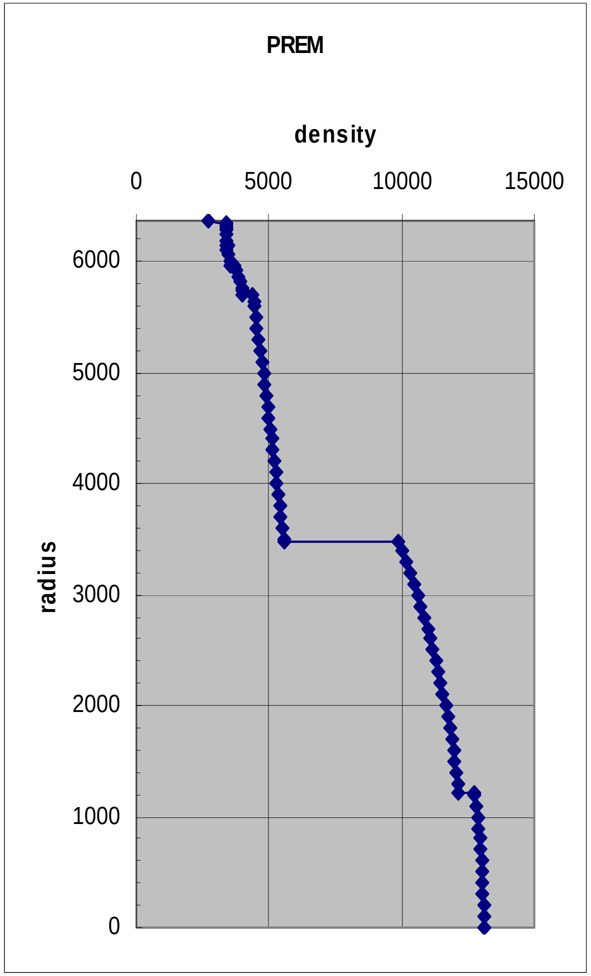


Equation of state (EoS) for different materials
Tests on solar system planets
Application to Extrasolar planets
Some questions to be solved like plate tectonics

Earth is unique in the solar system : water at its surface, life, plate tectonics, …
In Earth Science, one ongoing work is to understand the relationships between mantle convection and plate tectonics
Plate tectonics is important for the water cycle – Is it required for life to form and to develop?

Internal structure of the Earth





Mass = 6 10^{24} kg : 1/3 core and 2/3 mantle
Upper and Lower mantle
Subsolidus Convection in the mantle
Core : Iron + light element (S, O, other).
Mantle : $(Mg,Fe)_2Si_2O_6$, $Ca(Mg,Fe)Si_2O_6$, $(Mg,Fe)_2SiO_4$ and Al phase / $(Mg,Fe)SiO_3$, $(Mg,Fe)O$ and Al phase

Calculation of the radius

$$\frac{dP}{dr} = -\rho(r)g(r)$$

$$g(r) = \frac{4\pi G}{r^2}\int_0^r r'^2 \rho(r')dr'$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho C p}$$

$$P_{th} = \int_{T0}^{T} \alpha K_T \, dT$$

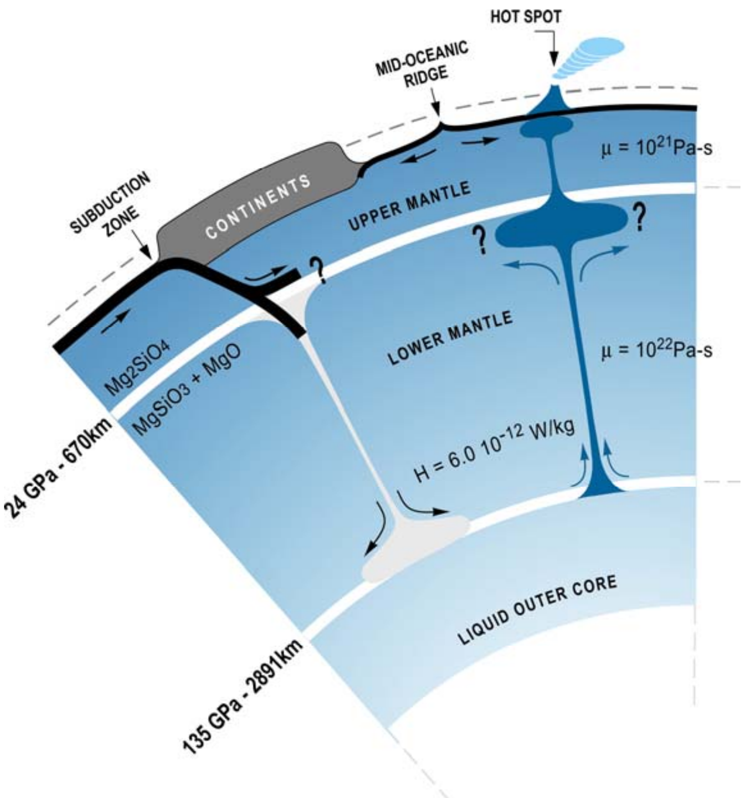
$$M = 4\pi \int_0^R r'^2 \rho(r')dr'$$

Mass is input parameter
Amount of H_2O is fixed
Main elements are O, Si, Mg, Fe
Fe is distributed between iron core and mantle
Fe# is the same in the two solid phases (varies from 0.1 to 0.3)
No inner solid core

We need an Equation of State (EoS) which relates density to pressure and temperature.
Example of the Birch-Murnagham EoS

$$P = \frac{3K_{0T}}{2}\left[\left(\frac{\rho}{\rho_0}\right)^{\frac{7}{3}} - \left(\frac{\rho}{\rho_0}\right)^{\frac{5}{3}}\right]\left\{1 + \frac{3}{4}(K'_{0T} - 4)\left[\left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}} - 1\right]\right\}$$

Internal structure of the Earth - composition



	EEH Earth model	PUM	LM	Core
O	30,28	44,76	43,8	1,61
Fe	33,39	5,89	12,69	80,25
Si	19,23	21,35	24,28	10,34
Mg	12,21	23,21	16,18	0
Total	**95,11**	**95,21**	**96,95**	**92,2**
Ni	2,02	0,25	0,71	4,99
Ca	1,01	2,32	1,2	0
Al	0,93	2,13	1,1	0
S	0,85	0,01	0,01	2,57
Total	**99,92**	**99,92**	**99,97**	**99,76**

O	30,28	44,76	43,8	1,61
Fe	35,41	6,14	13,4	85,24
Si	19,69	22,41	24,83	10,34
Mg	13,68	26,59	17,93	0

Core : Iron + light element (S, O, other).
Mantle : $(Mg,Fe)_2Si_2O_6$, $Ca(Mg,Fe)Si_2O_6$, $(Mg,Fe)_2SiO_4$ and Al phase / $(Mg,Fe)SiO_3$, $(Mg,Fe)O$ and Al phase

Input parameters

	EEH	PUM	LM	Core	2/3LM+1/3Core
Fe/Si	**0,909**	0,138	0,273	4,166	0,944
Mg/Si	**0,803**	1,372	0,835	0,000	0,691
Fe/(Fe+Mg)	0,531	0,092	0,246	1,000	0,577

Solar values		
	Mg,Fe,Si	+Ni,Ca,Al,S'
Fe/Si	0,977	**0,986**
Mg/Si	1,072	**1,131**
Fe/(Fe+Mg)	0,477	0,466

Five parameters are required:

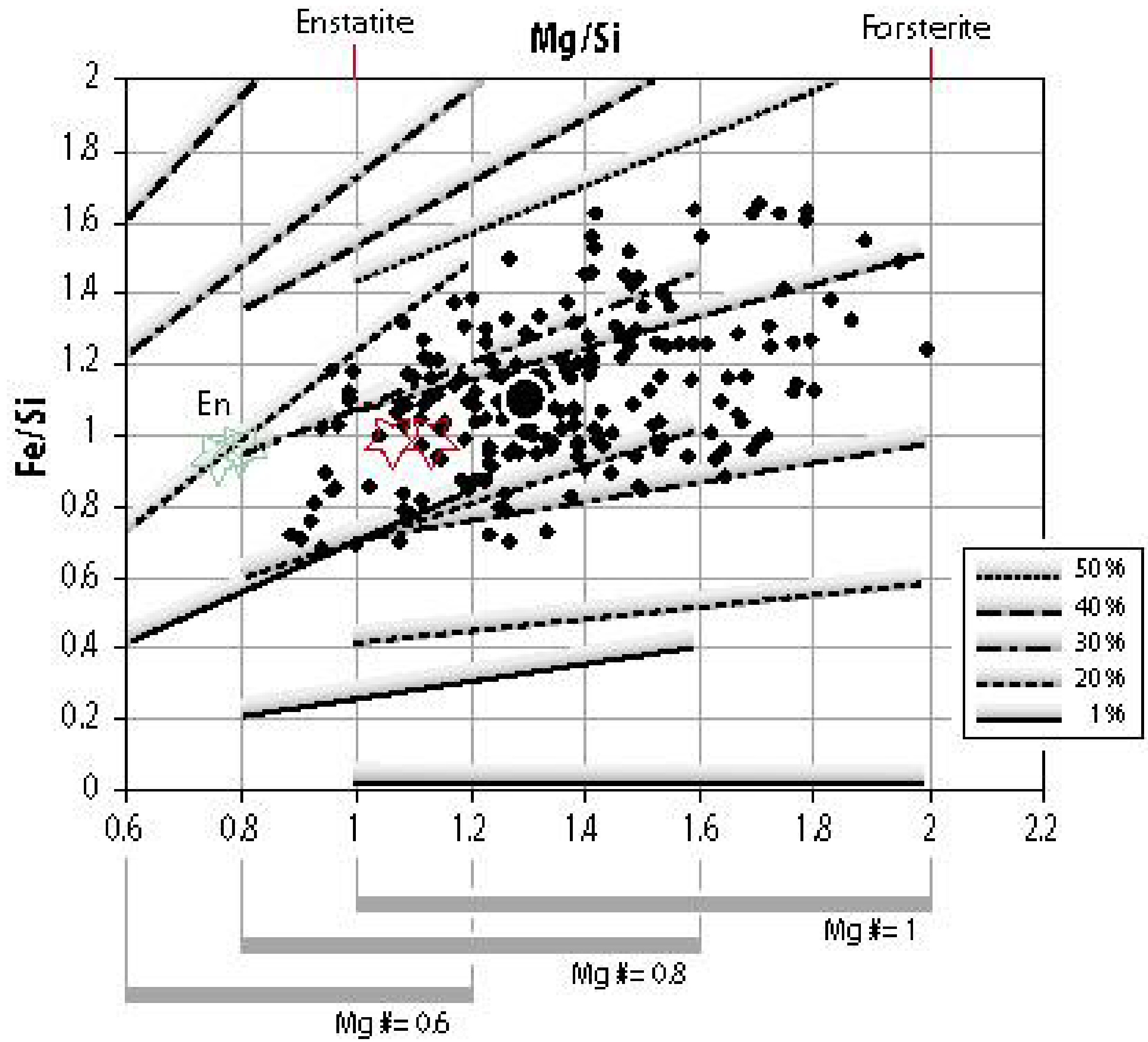
1) Total mass of the planet
2) Fe/Si (Stellar)
3) Mg/Si (Stellar)
4) Water mass fraction (Earth like / Ocean planet)
5) Mg# = Mg/(Mg+Fe)

Large uncertainties on the composition of the Earth – how does it influence the M(R) law

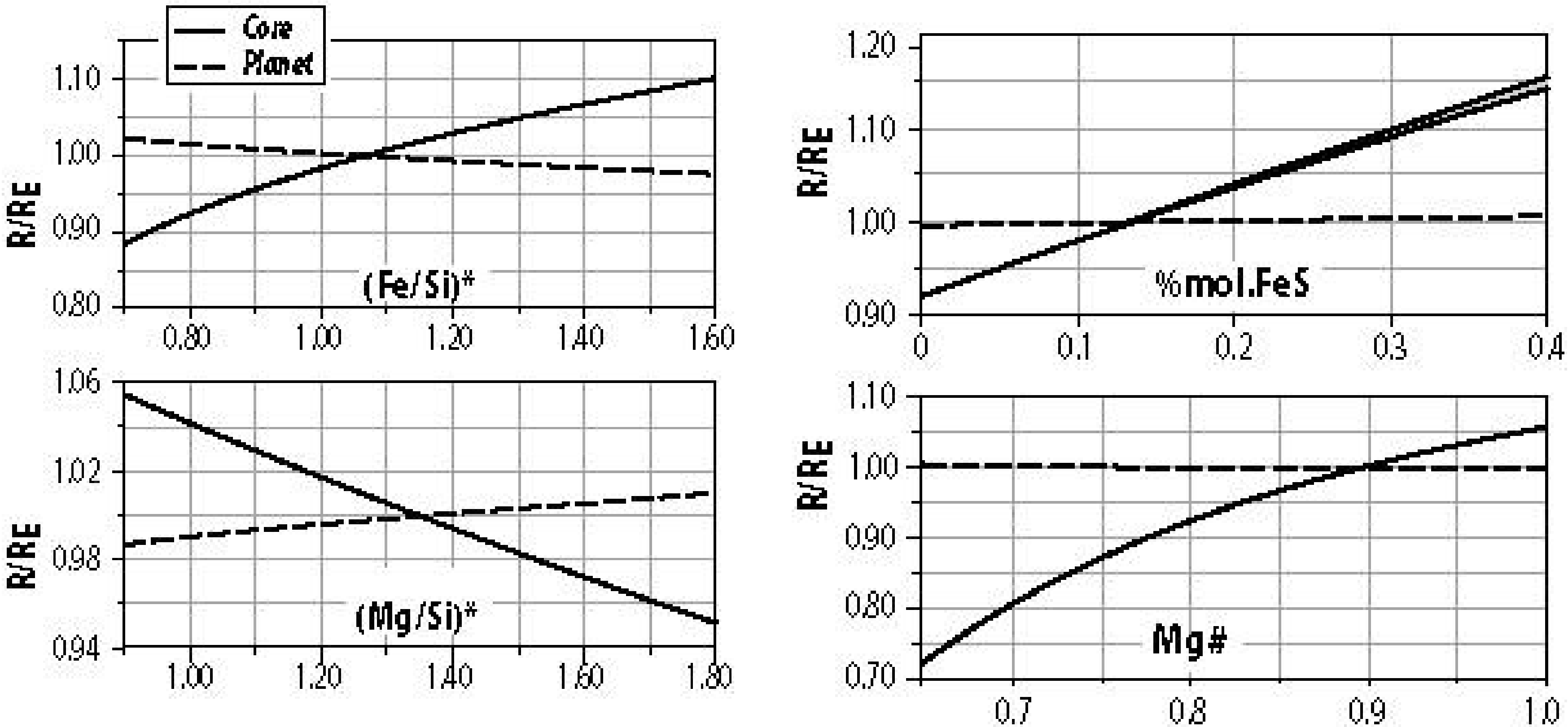
Composition of stars

Data from Beirao et al. (2006) and Gilli et al. (2006)

The difference between the composition of the planet compared to its star may be larger than the variability in composition of the family of stars.

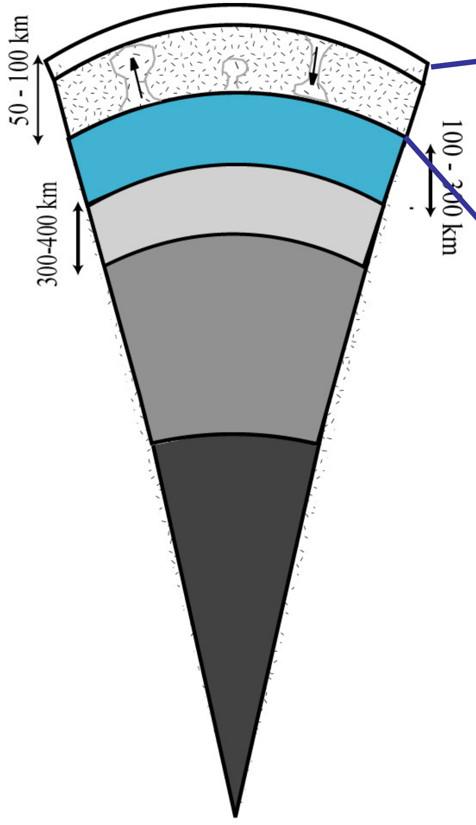


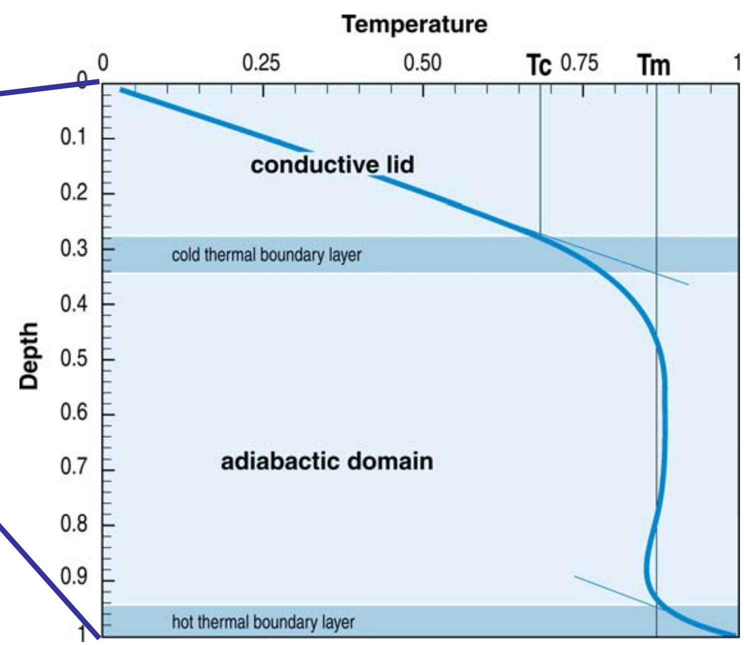
Radius versus composition (1 M_E < M < 10 M_E)



Total radius does not vary significantly with on the composition
The amount of Fe plays a significant role for the radius of the core

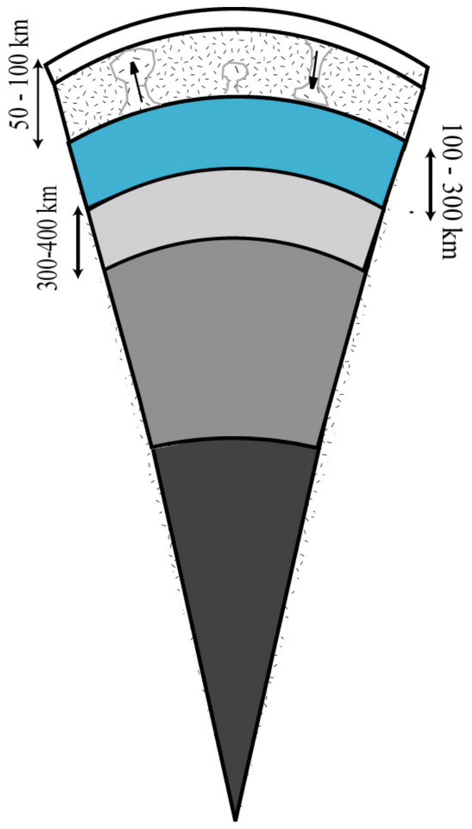
Internal structure of large icy satellites (1/2)

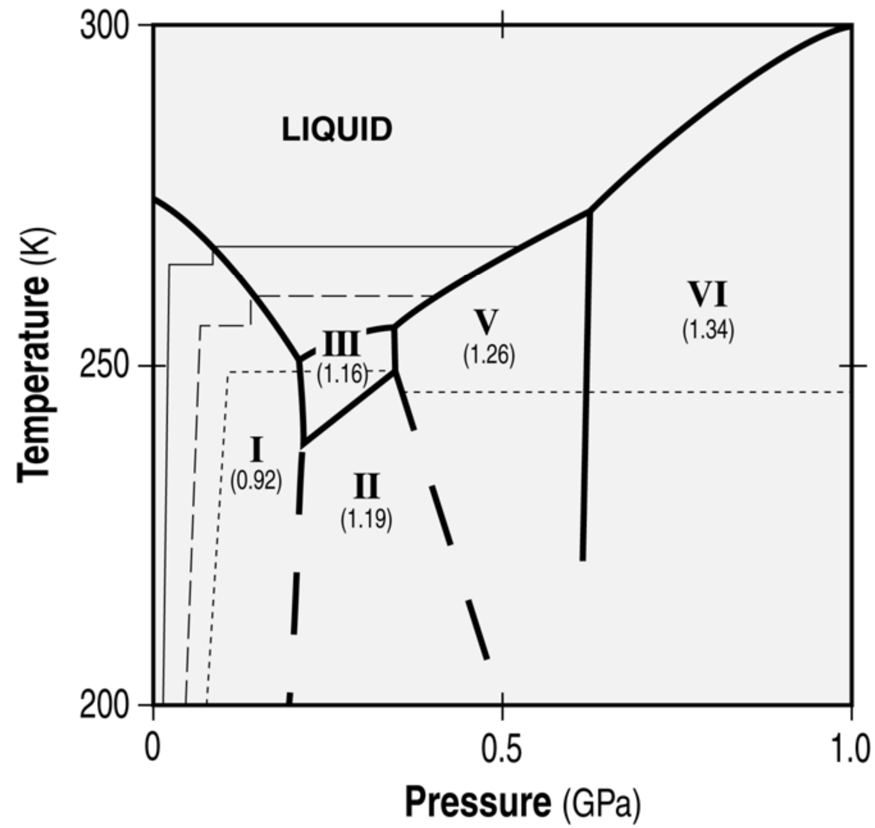




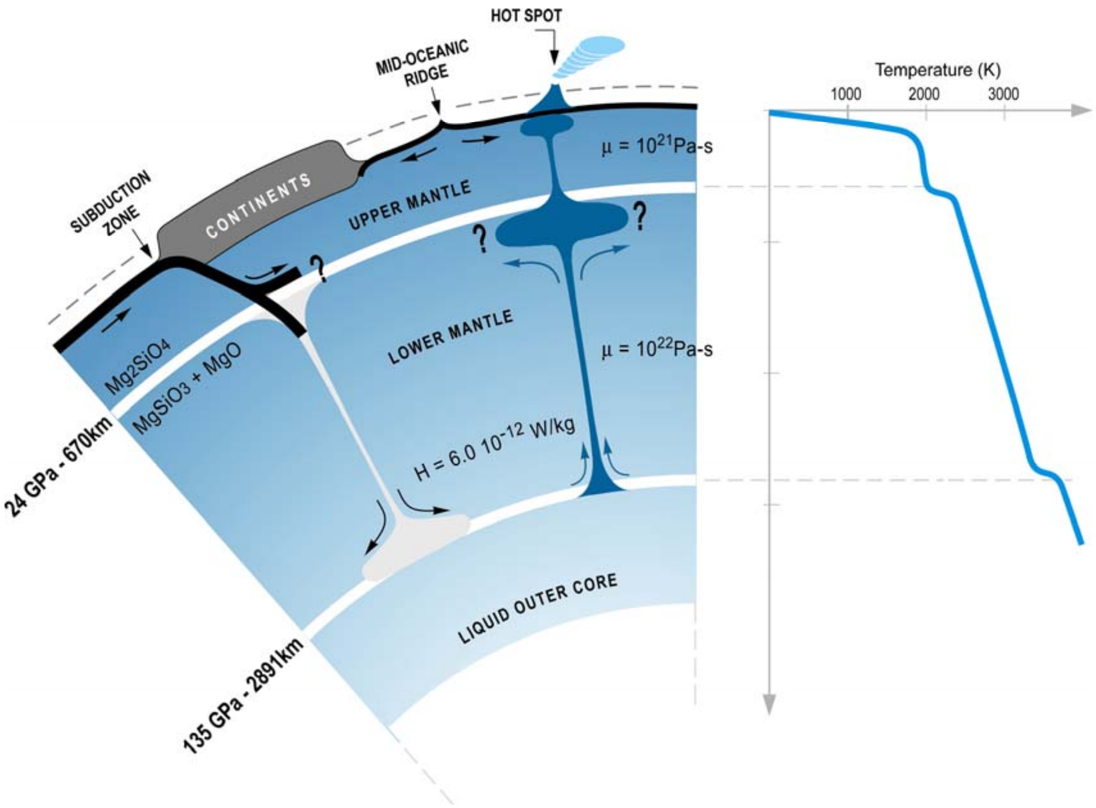
- Outer ice layer
- liquid layer
- High-pressure phases of ice
- Silicate layer
- Iron core

Internal structure of large icy satellites (2/2)





Modeling the mass - radius relationship. Temperature



$$\frac{dT}{dP} = \frac{\alpha T}{\rho C_P} = \frac{\gamma T}{\rho \Phi}$$

$$\Phi = \frac{K_S}{\rho} = \frac{dP}{d\rho}$$

$$\gamma = \gamma_0 \left(\frac{\rho_0}{\rho} \right)^q$$

T : Temperature
P : Pressure
ρ: density (kg/m^3)
α: thermal expansion coefficient (K^{-1})
C_p : heat capacity (J/kg/K)

Equation of State

Birch-Mürnhagan EOS

- Liquid layer
- Upper silicate mantle

$$
\begin{cases}
P(\rho,T) = \frac{3}{2}K_{T,0}^{0}\left[\left(\frac{\rho}{\rho_{T,0}}\right)^{7/3} - \left(\frac{\rho}{\rho_{T,0}}\right)^{5/3}\right]\left\{1-\frac{3}{4}\left(4-K_{T,0}^{'}\right)\left[\left(\frac{\rho}{\rho_{T,0}}\right)^{2/3}-1\right]\right\} \\
K_{T,0}^{0} = K_0 + a_P(T-T_0) \\
K_{T,0}^{'} = K_0^{'} \\
\rho_{T,0} = \rho_0 \exp\left(\int_{300}^{T}\alpha_{T,0}dT\right) \\
\alpha_{T,0} = a_T + b_T.T - c_T.T^{-2}
\end{cases}
$$

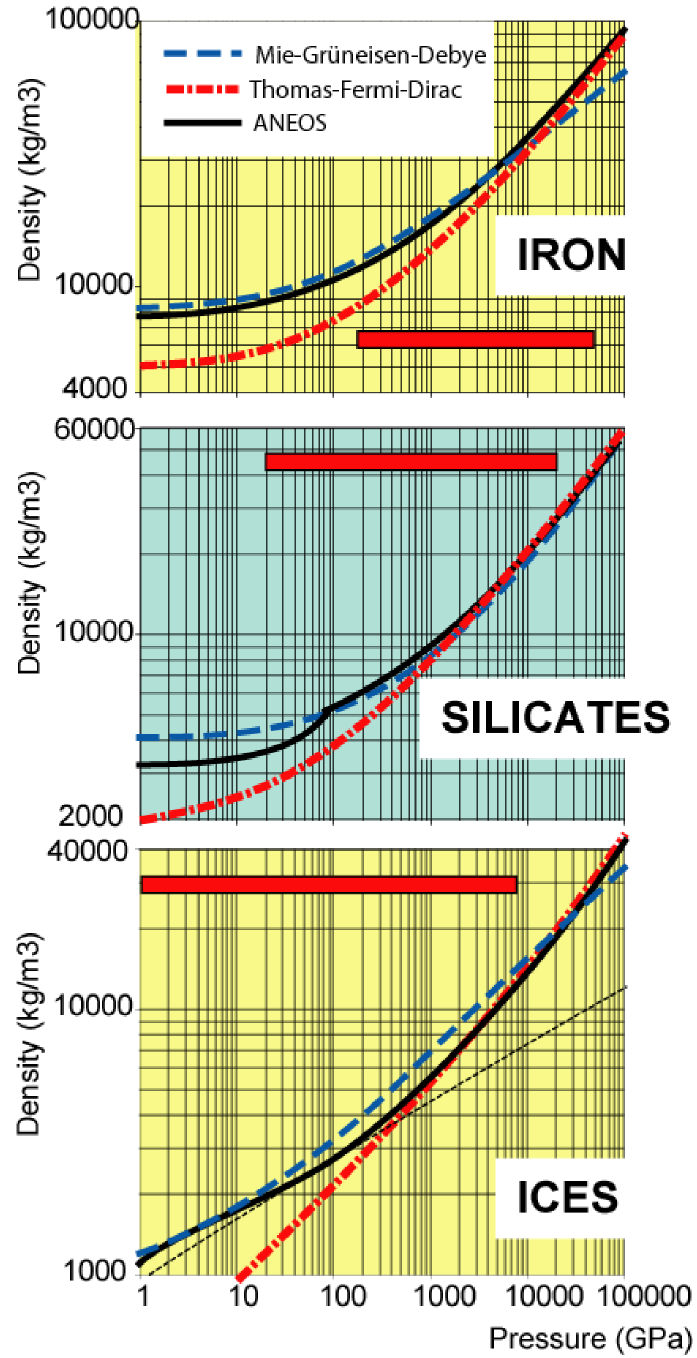
Mie-Grüneisen-Debye EOS

- Lower silicate mantle

This approach dissociates static pressure and thermal pressure

Thomas-Fermi-Dirac

- Icy mantle
- Metallic core (P> 1 TPa)



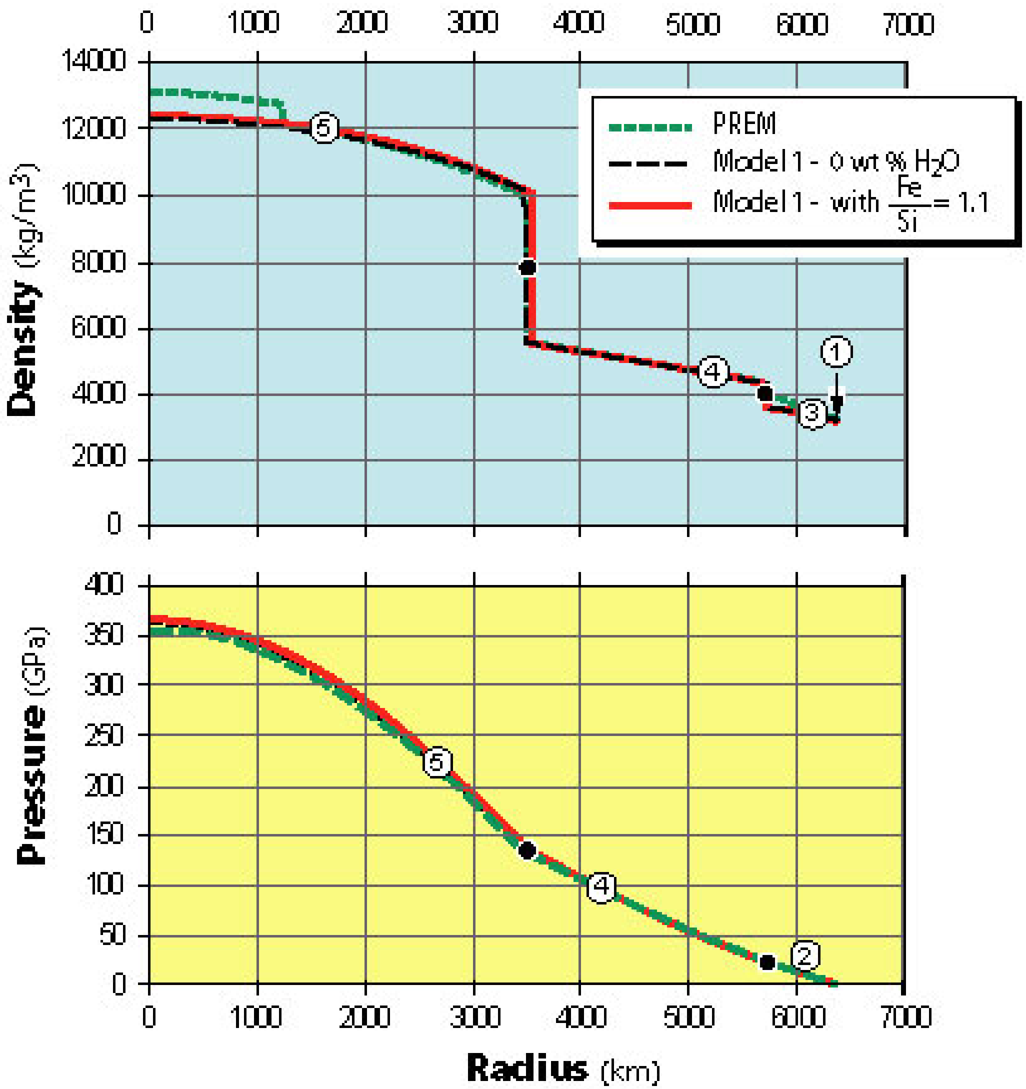
Results : Validation of the model - Earth

<u>Model</u> :

- $Fe/Si = 0.987$
- $Mg/Si = 1.136$
- $Mg\# = 0.9$
- H_2O: 0.01 wt %

$M=M_{Earth}$

R=6414 km (0.6%)

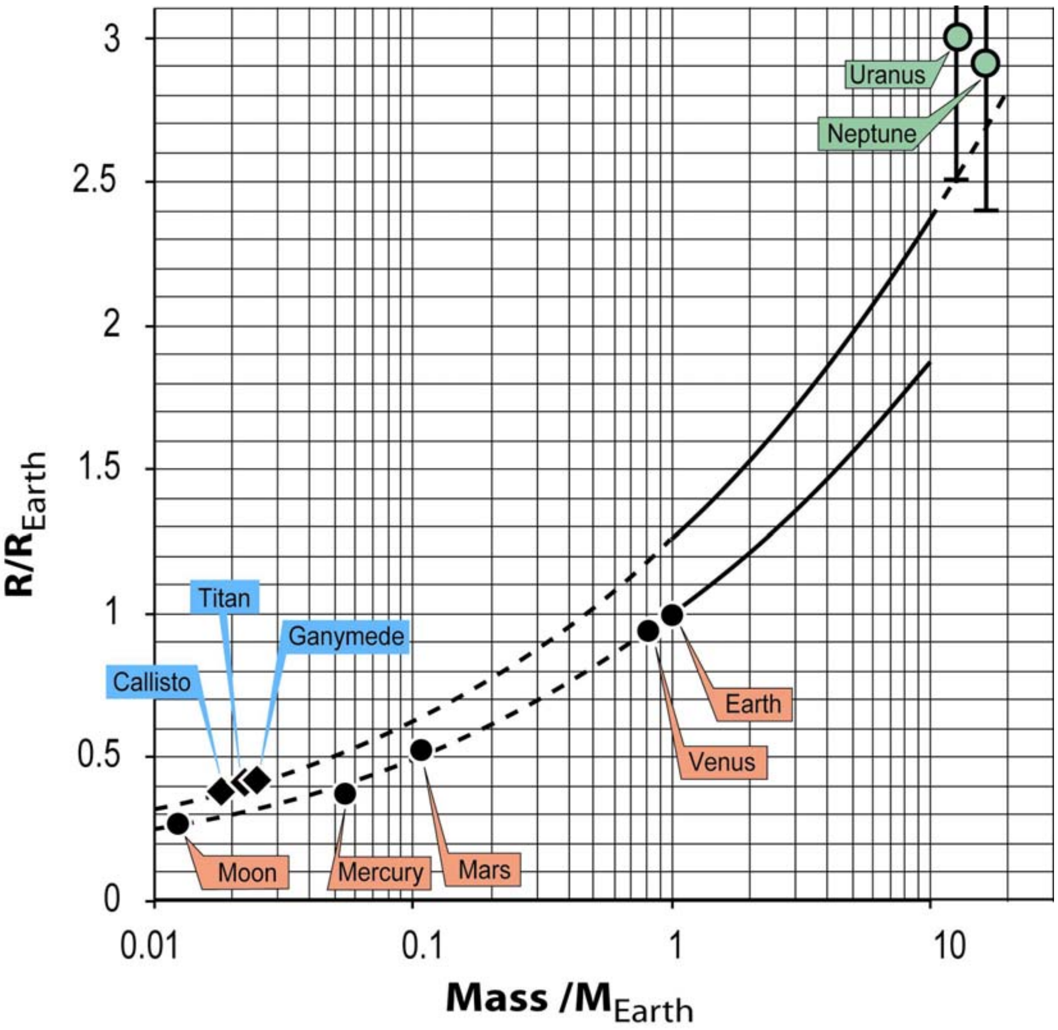


Results : Validation of the model – Solar system

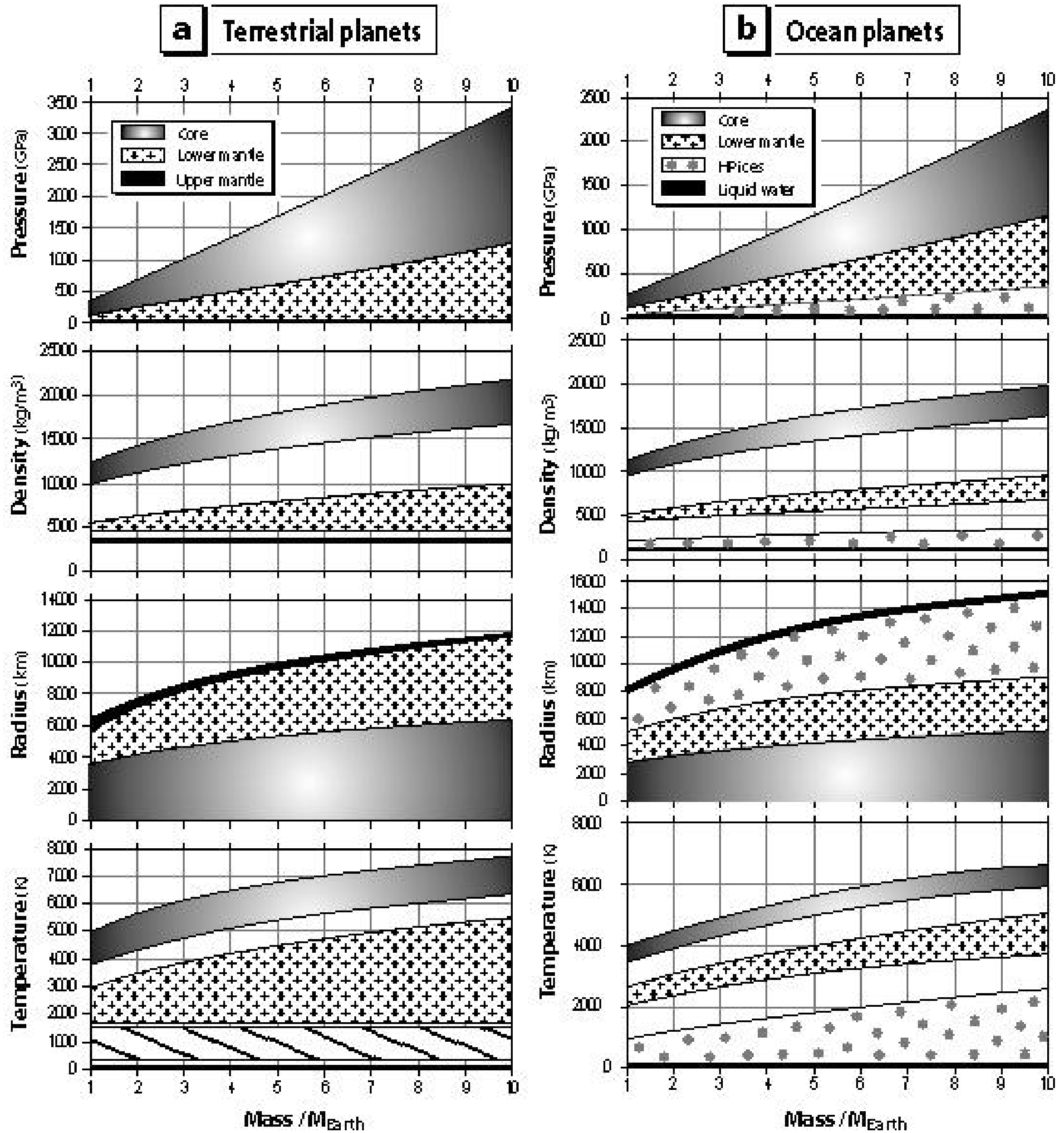
$$\frac{R}{R_{Earth}} = \left(\frac{M}{M_{Earth}}\right)^{0.274}$$

	Earth-like		Ocean/Icy	
0.01-1	1.00	0.306	1.258	0.302
1-10	1.00	0.274	1.262	0.275

A planet with 50% water is 26% larger than a planet without water (for the same total mass)



Results : Variations with mass of different parameters



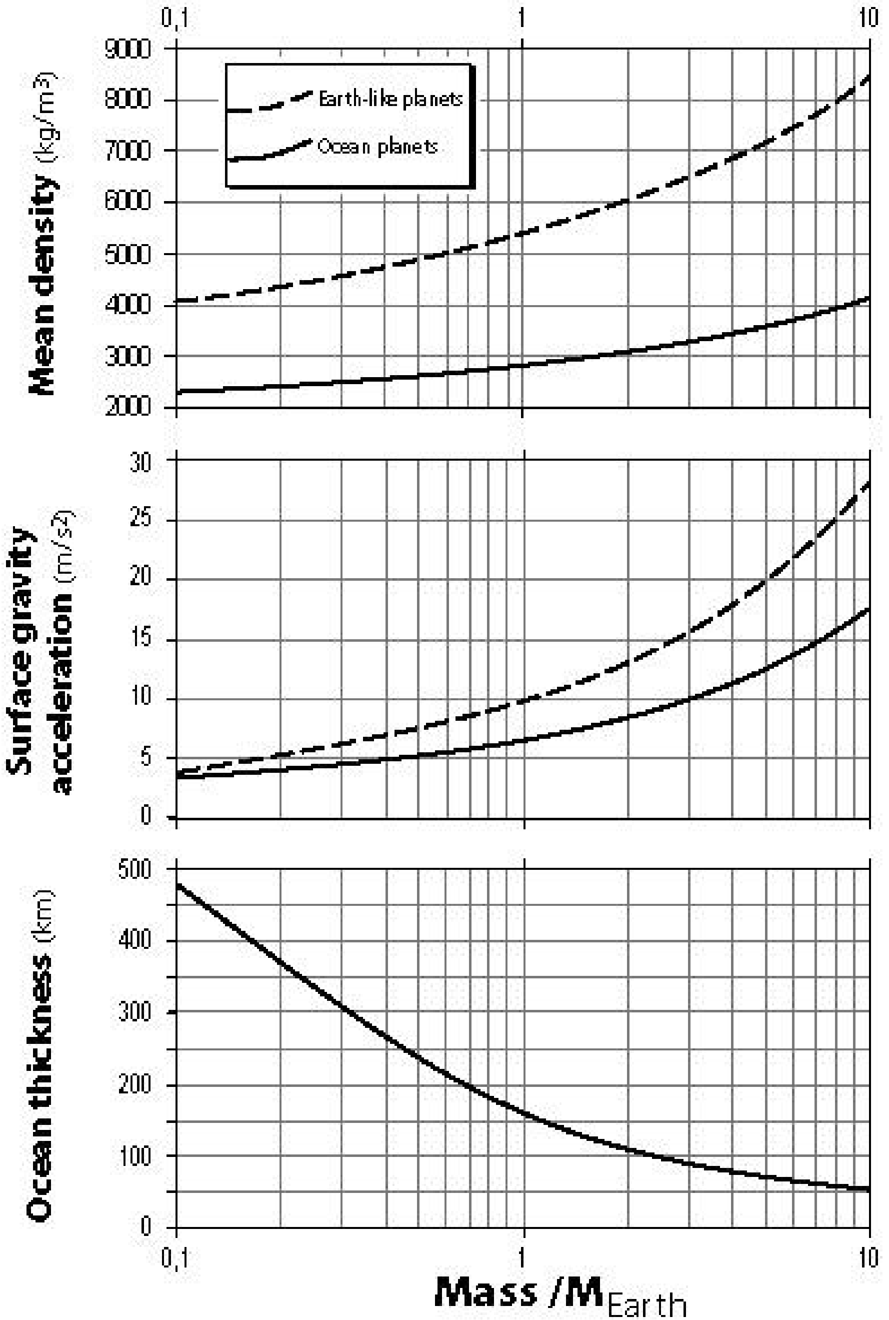
Pressure

Density

Radius

Temperature

Results : Variations with mass of different parameters



Mean density

Surface gravity

Ocean thickness

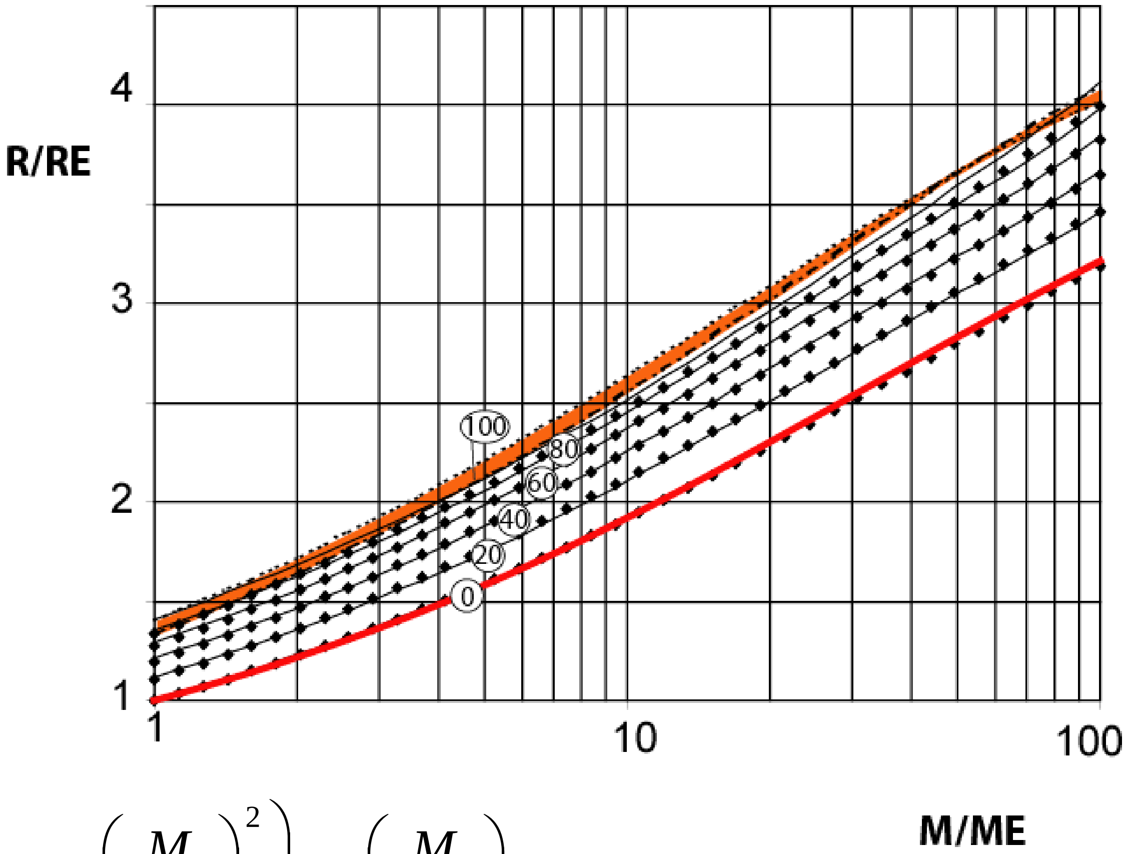
Results : Extrapolation to larger planets

$$\frac{R}{R_{Earth}} = \left(\frac{M}{M_{Earth}}\right)^{0.274}$$

Reference Case :

- **Fe/Si = 1.10 ***
- **Mg/Si = 1.25 ***
- **Mg# = 0.8**
- **H_2O: 0.01 wt %**

* Averaged from Gilli et al., A&A, 2006

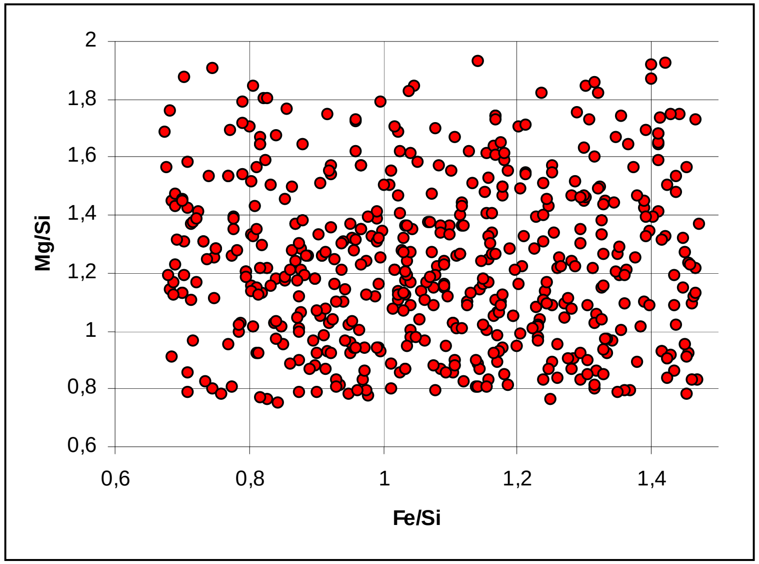


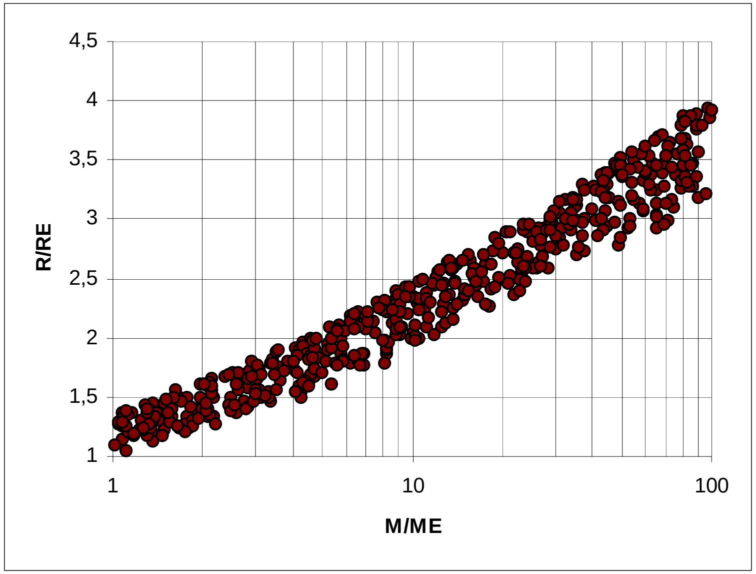
$$\log\left(\frac{R}{R_E}\right) = \log(\alpha) + \left(\beta + \gamma\frac{M}{M_E} + \varepsilon\left(\frac{M}{M_E}\right)^2\right)\log\left(\frac{M}{M_E}\right)$$

Each coefficient depends on the amount of water (X)

$$\xi = \sum_{i=0}^{2} \xi_i \, X_w^{i-1}$$

Results : Uncertainties (1/3)





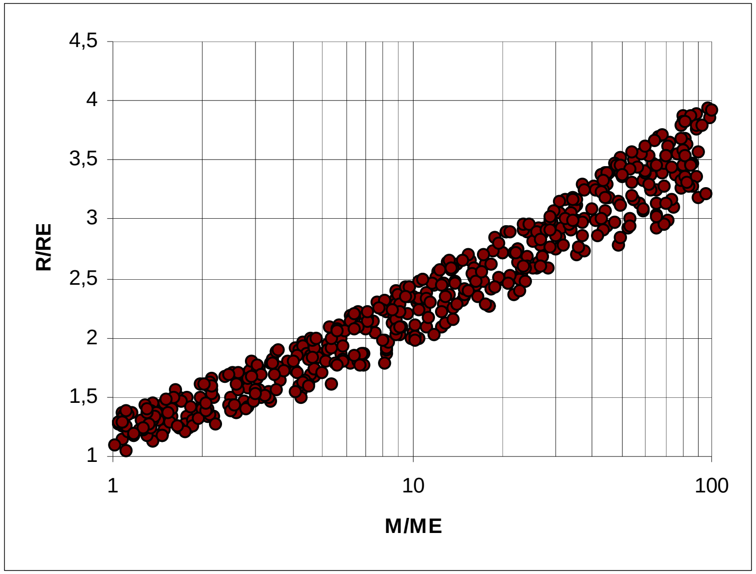
530 simulations

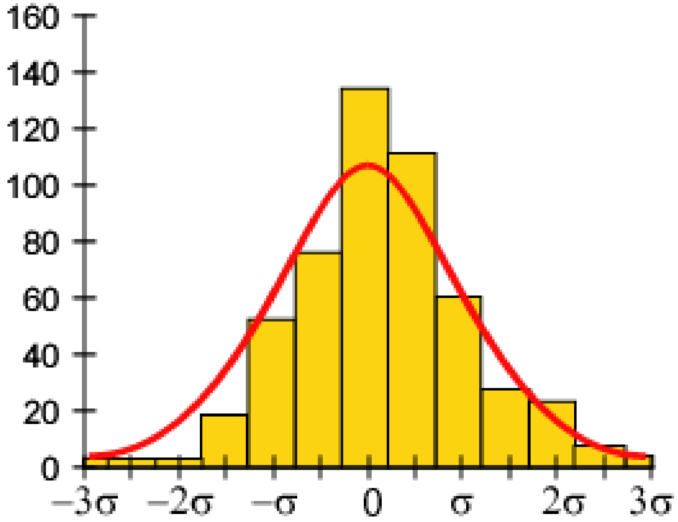
Variables randomly distributed

- **Fe/Si**
- **Mg/Si**
- **Mg#**
- **H_2O**

 ΔT through interfaces

Results : Uncertainties (2/3)





If mass and radius are exactly known, the amount of water can be estimated with an accuracy of $\sigma = \pm 4.5\ \%$

Results : Uncertainties (3/3)

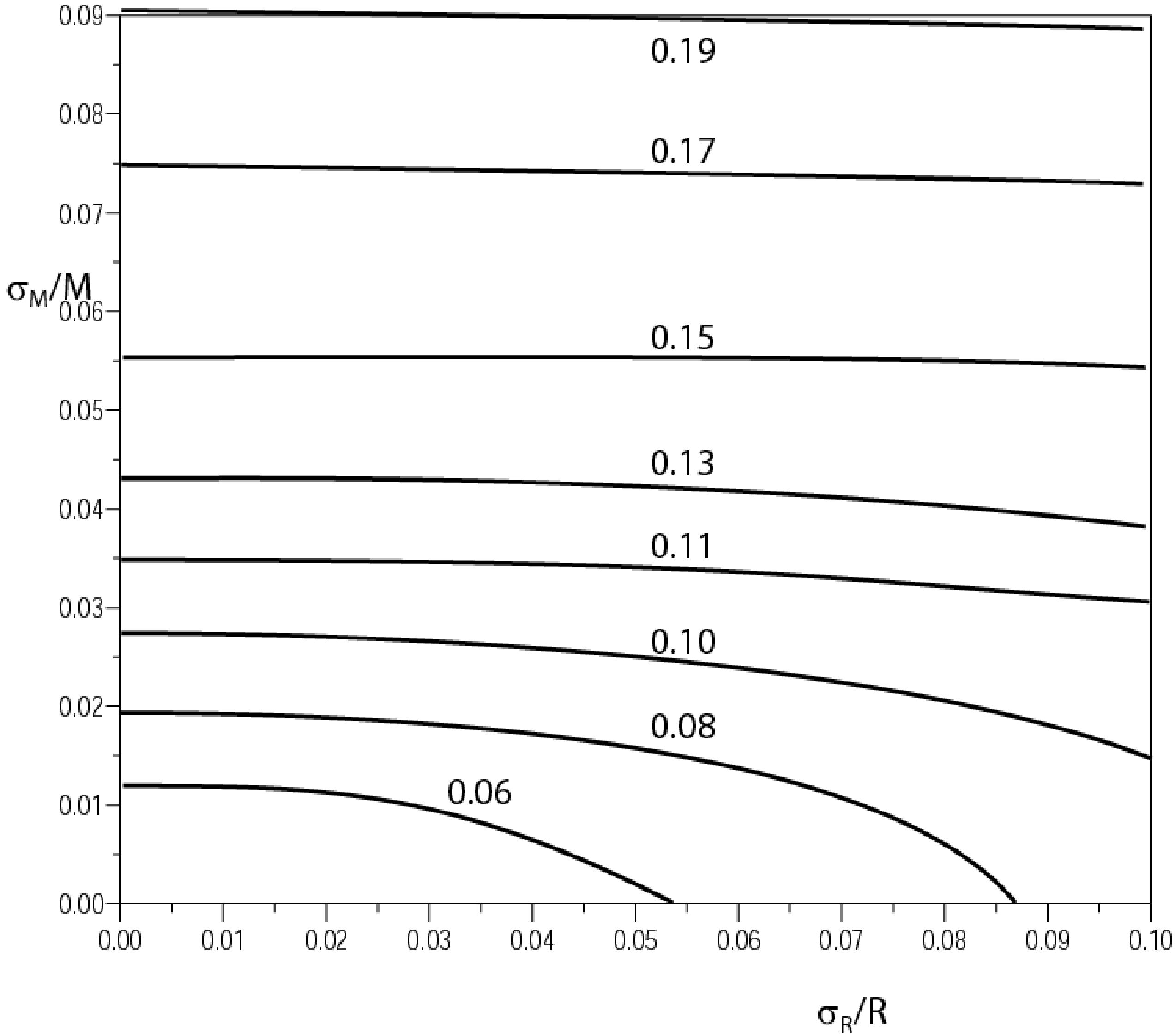


Plate tectonics on large Earths

Plate tectonics provides a recycle of volatiles on geological timescales that may be important for the development of life

Two papers came out at the same time with two different conclusions

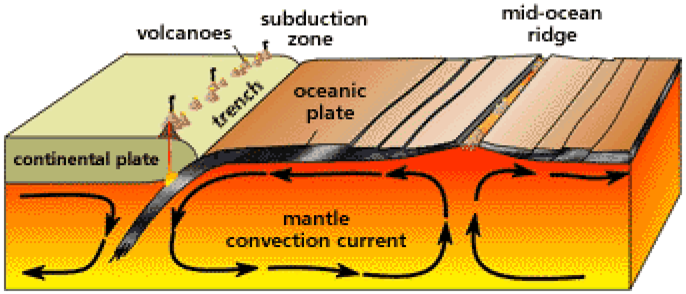
Valencia et al., ApJ, 2007

O'Neill and Lenardic, GRL, 2007

Valencia et al. : …. We demonstrate that as **planetary mass increases**, the **shear stress** available to overcome resistance to plate motion **increases** while the **plate thickness decreases**, thereby enhancing plate weakness.
These effects contribute favorably to the subduction of the lithosphere, an essential component of plate tectonics.
Moreover, uncertainties in achieving plate tectonics in the one earth-mass regime disappear as mass increases: super-Earths, even if dry, will exhibit plate tectonic behavior.

O'Neil and Lenardic : … mantle convection simulations have been carried out to show that simply **increasing planetary radius** acts to **decrease** the ratio of driving to resisting stresses, and thus super-sized Earths are likely to be **in an episodic or stagnant lid regime**.

Calculation of lithosphere thickness and stress



The deviatoric horizontal normal stress (σ) responsible for causing failure on the plate is (to first order) balanced by the shear stress (τ) applied over the base of the plate.

$$\sigma \approx \tau \frac{L}{\delta}$$

The thickness of the lithosphere or boundary layer (δ) depends on the Rayleigh number (Ra) – a parameter governing convection.

$$\frac{\delta}{D} = \left(\frac{Ra}{Ra_c}\right)^S$$

With S=-1/4

$$Ra = \frac{\alpha \rho g D^4 q}{k \kappa \eta}$$

$$\tau \approx \eta \frac{u}{D}$$

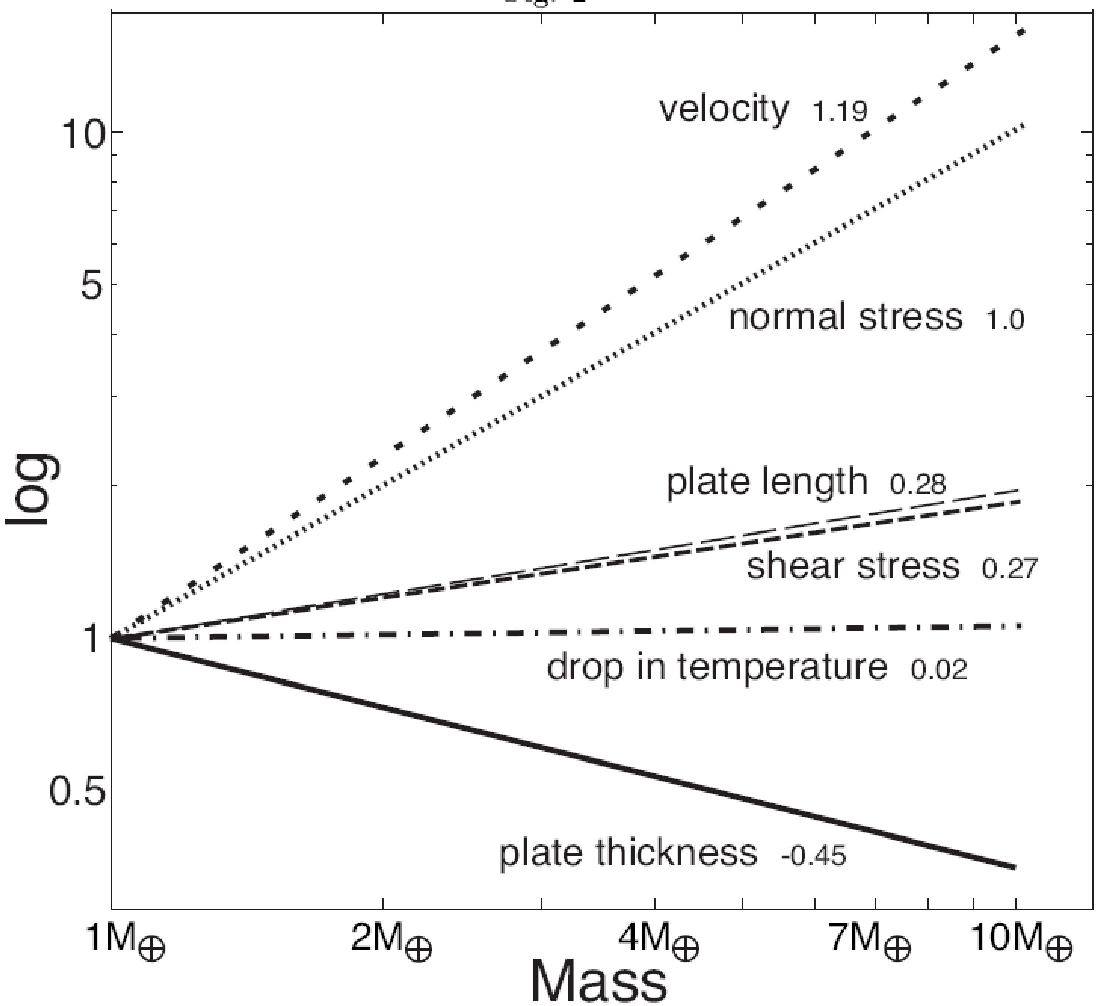
$$u \approx \frac{\kappa}{D}(Ra)^{0.5}$$

q is heat flux (radiogenic and cooling rate)

τ is the deviatoric stress
η is viscosity (Pa.s)
U is velocity
D is thickness of the convective layer

Results (Valencia et al.)

Fig. 2



$$Ra = \frac{\alpha\rho g D^4 q}{k\kappa\eta}$$

$$\frac{\rho}{\rho_{Earth}} = \left(\frac{M}{M_{Earth}}\right)^{0.2}$$

$$\frac{g}{g_{Earth}} = \left(\frac{M}{M_{Earth}}\right)^{0.5}$$

$$\frac{D}{D_{Earth}} = \left(\frac{M}{M_{Earth}}\right)^{0.28}$$

q is proportional to M

$$u \approx M^{1.19}$$

and $\tau \approx M^{0.27}$

Results

$$\tau \approx \eta \frac{u}{D}$$

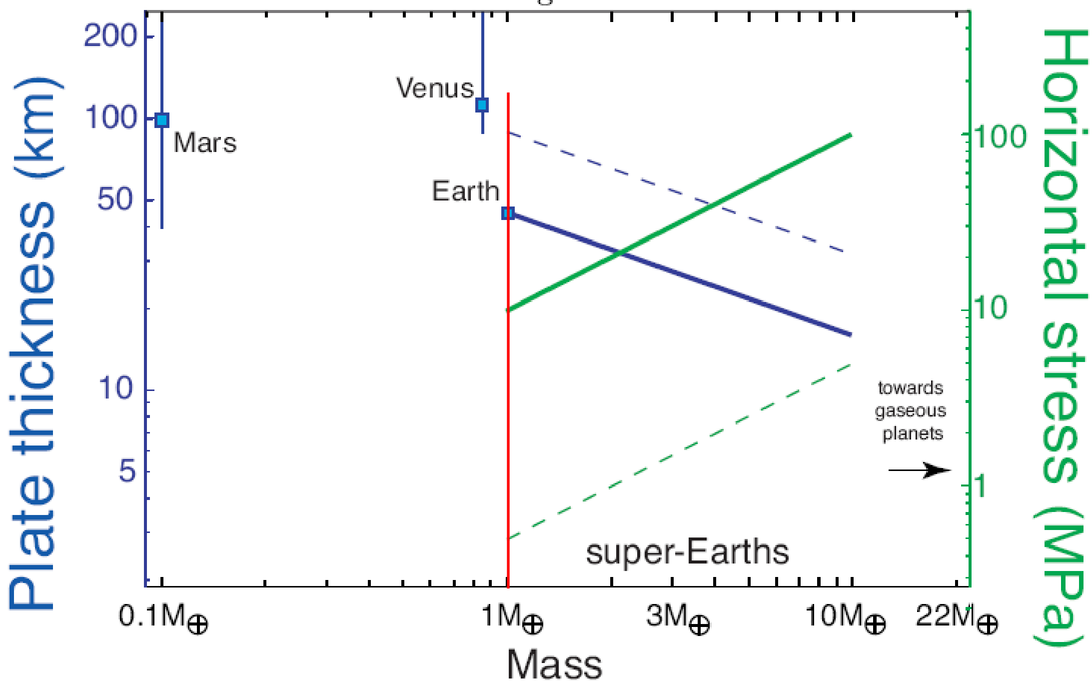
$$Ra = \frac{\alpha \rho g D^4 q}{k \kappa \eta}$$

$$\frac{\rho}{\rho_{Earth}} = \left(\frac{M}{M_{Earth}} \right)^{0.2}$$

$$\frac{g}{g_{Earth}} = \left(\frac{M}{M_{Earth}} \right)^{0.5}$$

$$\frac{D}{D_{Earth}} = \left(\frac{M}{M_{Earth}} \right)^{0.28}$$

Fig. 1

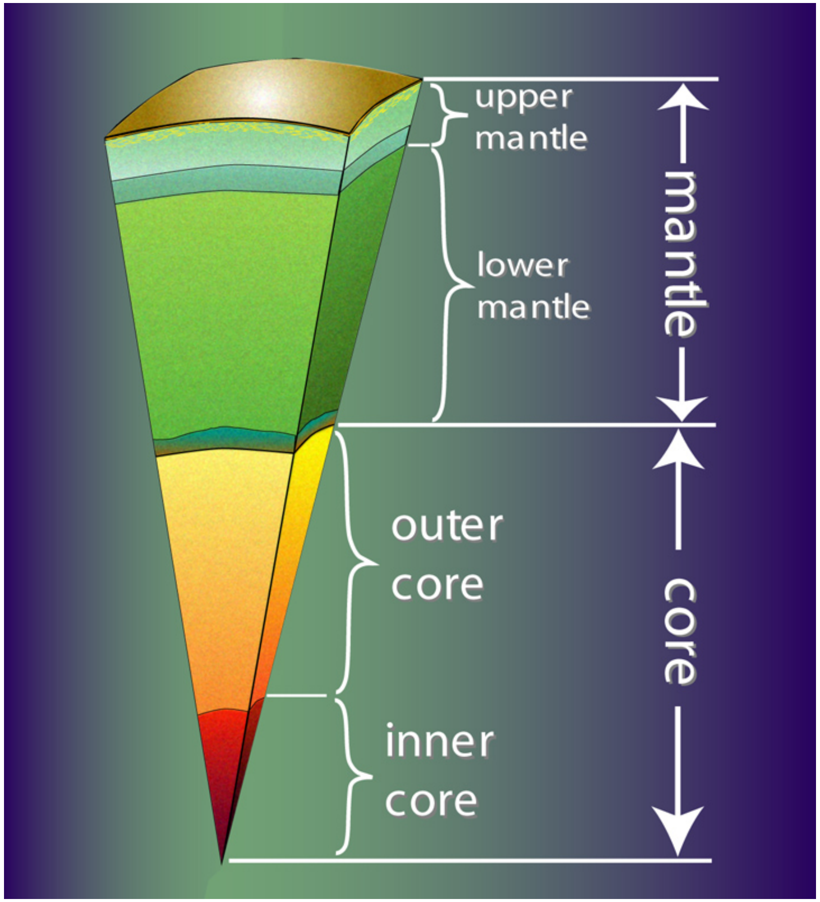


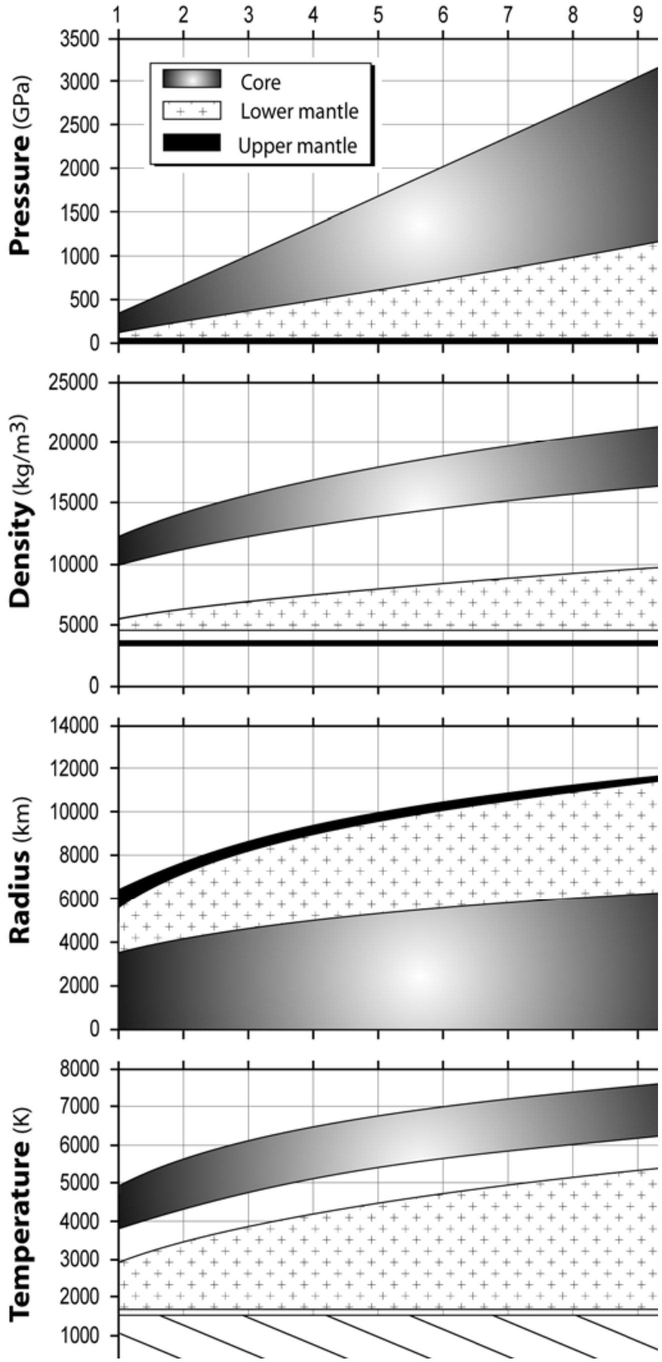
q is proportional to M

$$\delta \approx M^{-0.45} \quad \text{and} \quad \sigma \approx M^{1.0}$$

Given that Earth's convective state leads to plate tectonics, the more favorable conditions experienced by super-Earths will inevitably lead to plate tectonics.

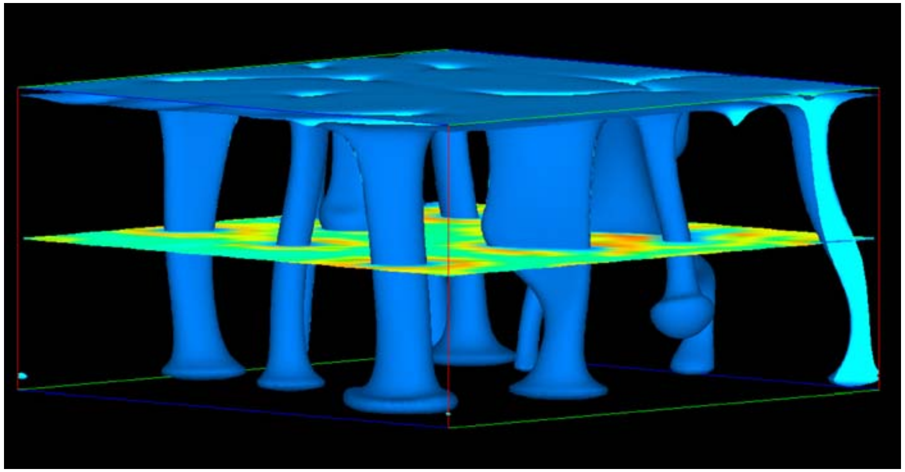
Discussion : upper mantle – lower mantle barrier to convection

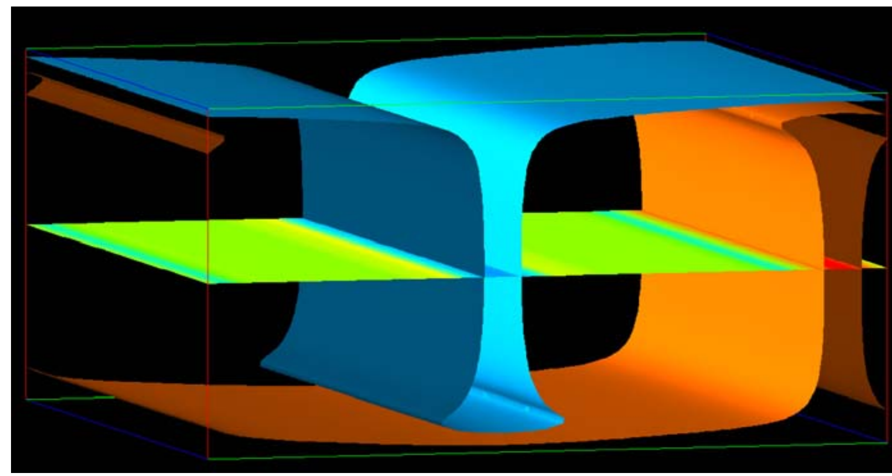


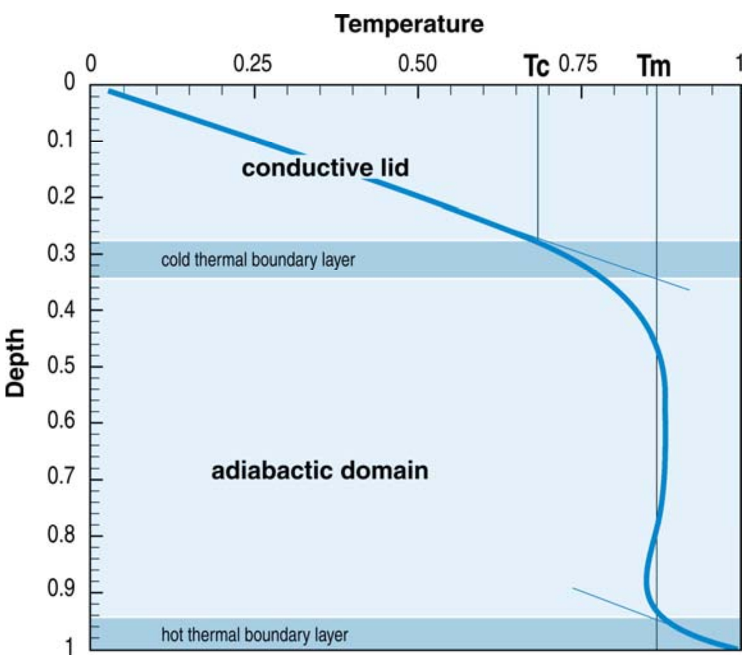


Thickness of the upper mantle decreases with mass. Transition around 23 GPa. Ra may decrease.

Discussion : transition from stagnant lid convection to plate tectonics confusion between lithosphere thickness and thermal boundary layer thickness in a convective 'mantle fluid'

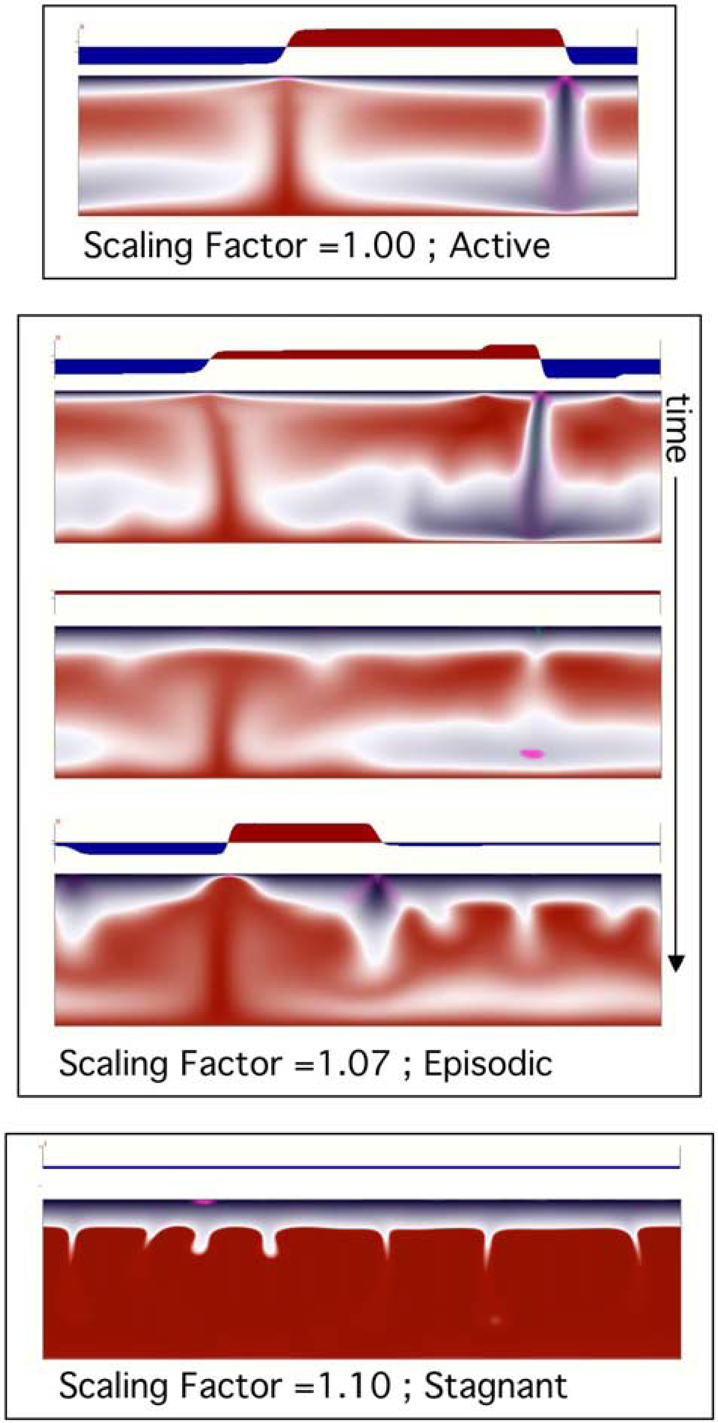






$$\frac{\delta}{D} = \left(\frac{Ra}{Ra_c}\right)^S$$

δ depends on Ra (according to laboratory experiments and numerical studies)



“Geological consequences of super-sized Earths”

by C. O’Neill and A. Lenardic

mantle convection simulations have been carried out to show that simply **increasing planetary radius** acts to **decrease** the ratio of driving to resisting stresses, and thus super-sized Earths are likely to be **in an episodic or stagnant lid regime**.

$$\frac{\tau}{\sigma}\downarrow$$

"Geological consequences of super-sized Earths"

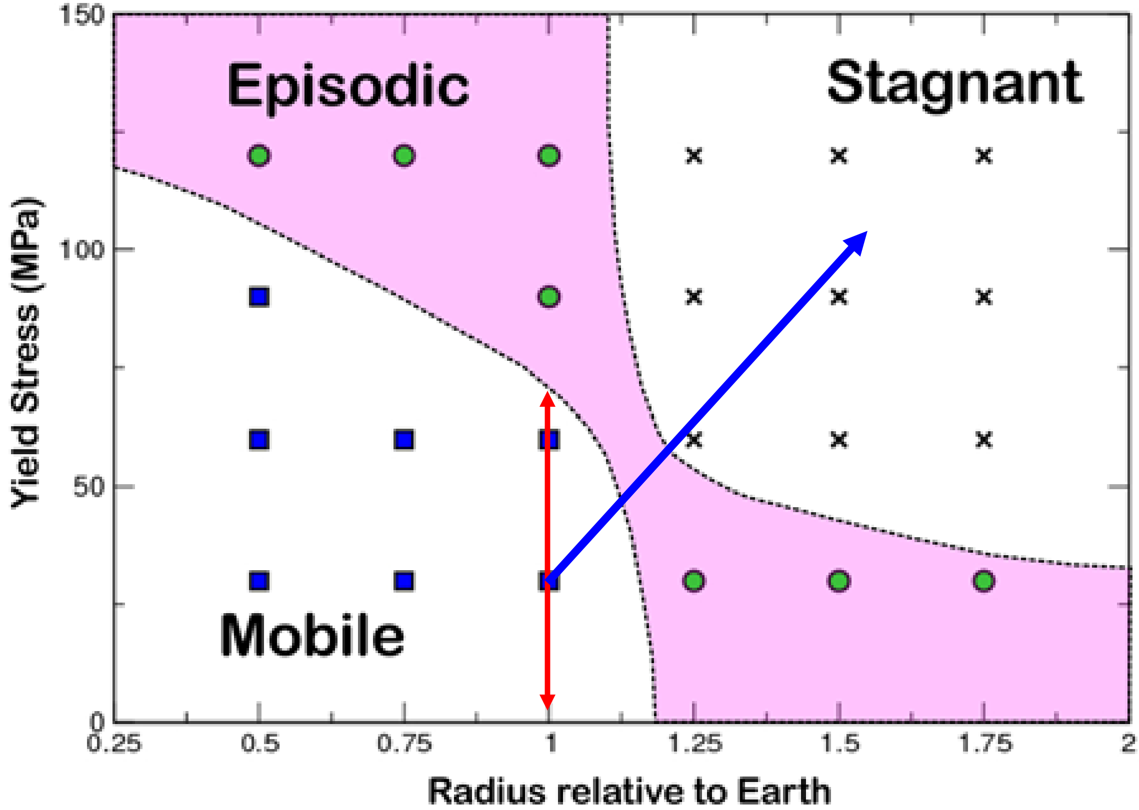
by C. O'Neill and A. Lenardic

$$\sigma \approx \tau \frac{L}{\delta}$$

$$\tau \approx \eta \frac{u}{D}$$

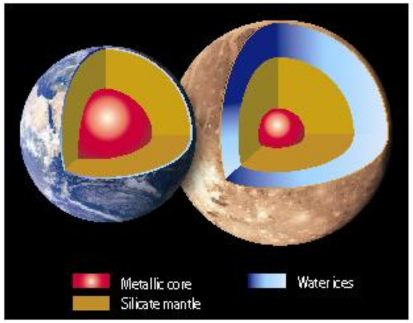
$$\tau \approx M^{0.27}$$

$$u \approx M^{1.19}$$



Why is there no plate tectonics on Mars and Venus? Water?

Conclusions



- Very good prediction of radii
- Amount of water is a first order parameter
- **Radius is 26 % larger for an Ocean planet with 50 %wt of ices**
- Temperature is a second order parameters.
- Composition and Mg# control the size of the core.
- **If Mass and Radius are perfectly known, the amount of water can be known at ± 4.4 %**
- **If 10% uncertainty of mass and radius, then the amount of water can be known at ± 20 %**
- Planetary classification seems possible with missions like COROT and KEPLER for measuring radius and instruments like HARPS for measuring the mass. Statistics will be possible. Atmospheric composition is an important parameter.
- The question of plate tectonics is open and controversial. Different parameters (including the presence of water, the basalt/eclogite transition) need to be studied. Plate tectonics provides much more volcanism than one plate planet.
- Upcoming work will address the transition between Earth-like and gaseous planets